Q.P. Code – 50722

Second Year B.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

(DSB 230) Paper II – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing atleast **ONE** from each Part.*

PART – A

1. (a) (i) Find the order and degree of differential equation $\frac{d^2y}{dx^2}+a^2x=0$. **2**

(ii) Show that $y=a\cos x+b\sin x$ is the solution of the differential equation $\frac{d^2y}{dx^2}+y=0$. **2**

(b) Solve $(x^2-y^2)dx=2xy\,dy$. **5**

(c) Solve $(2xy^2-y)dx+x\,dy=0$. **6**

2. (a) (i) Solve $P^2+2Px-3x^2=0$. **2**

(ii) Find the general and singular solution of the equation $y=Px+\frac{a}{P}$. **2**

(b) Solve $16x^2+2P^2y-P^3x=0$. **5**

(c) Show that the family of parabolas $y^2=4a(x+a)$ is self orthogonal. **6**

PART – B

3. (a) (i) Solve $\left[D^2+8D+16\right]y=0$. **2**

(ii) Solve $\left[D^4+8D^2+16\right]y=\cos 3x+5$. **2**

(b) Solve $\left(D^2-2D+5\right)y=e^x\cos 2x$. **5**

(c) Solve the simultaneous equations

$$(D+7)x-y=0$$
$$2x+(D+5)y=0$$ **6**

4. (a) (i) Evaluate $\lim_{x\to 1}\left[\frac{2}{x^2-1}-\frac{1}{x-1}\right]$. **2**

(ii) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$. **2**

(b) Expand $\log(1+\sin x)$ upto x^4 using Maclaurin's series. **5**

(c) State and prove Roll's theorem. **6**

PART – C

5. (a) (i) In a group G, if every element has its own inverse then prove that G is abelian. **2**

(ii) Find the number of generators of the cyclic group of order 30. **2**

(b) Prove that in a group G, $o(a)=o\left(a^{-1}\right)\ \forall\ a\in G$. **5**

(c) State and prove Euler's theorem. **6**

6. (a) (i) Solve $2x-3<5x+3<2x+3$. **2**

(ii) For any two real numbers x and y show that $|x+y|\le|x|+|y|$. **2**

(b) Find the order of the permutation φ and also find whether it is even or odd, where $\varphi=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10\\5&9&6&3&1&4&2&10&8&7\end{pmatrix}$. **5**

(c) Find the envelope of the family of lines $x\cos^3\alpha+y\sin^3\alpha=a$, where α is a parameter. **6**

PART – D

7. (a) (i) Find the limit of the sequence $\frac{2n+3}{5n-4}$. **2**

(ii) Show that the sequence $\{x_n\} = n(n+1)$ is monotonic. **2**

(b) If the sequence $\{x_n\}$ converges to l and $\{y_n\}$ converges to m then show that $\{x_n + y_n\}$ converges to $l+m$. **5**

(c) Prove that every convergent sequence is bounded. **6**

8. (a) (i) Show that the series $\sum \frac{1}{n(n+1)}$ is converges to 1. **2**

(ii) Discuss the convergence of the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$. **2**

(b) State and prove D'Alemberts ratio test. **5**

(c) Find the sum to infinity of the series

$$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 5}+\frac{1}{3\cdot 7}+\cdots \text{to}\,\infty$$ **6**